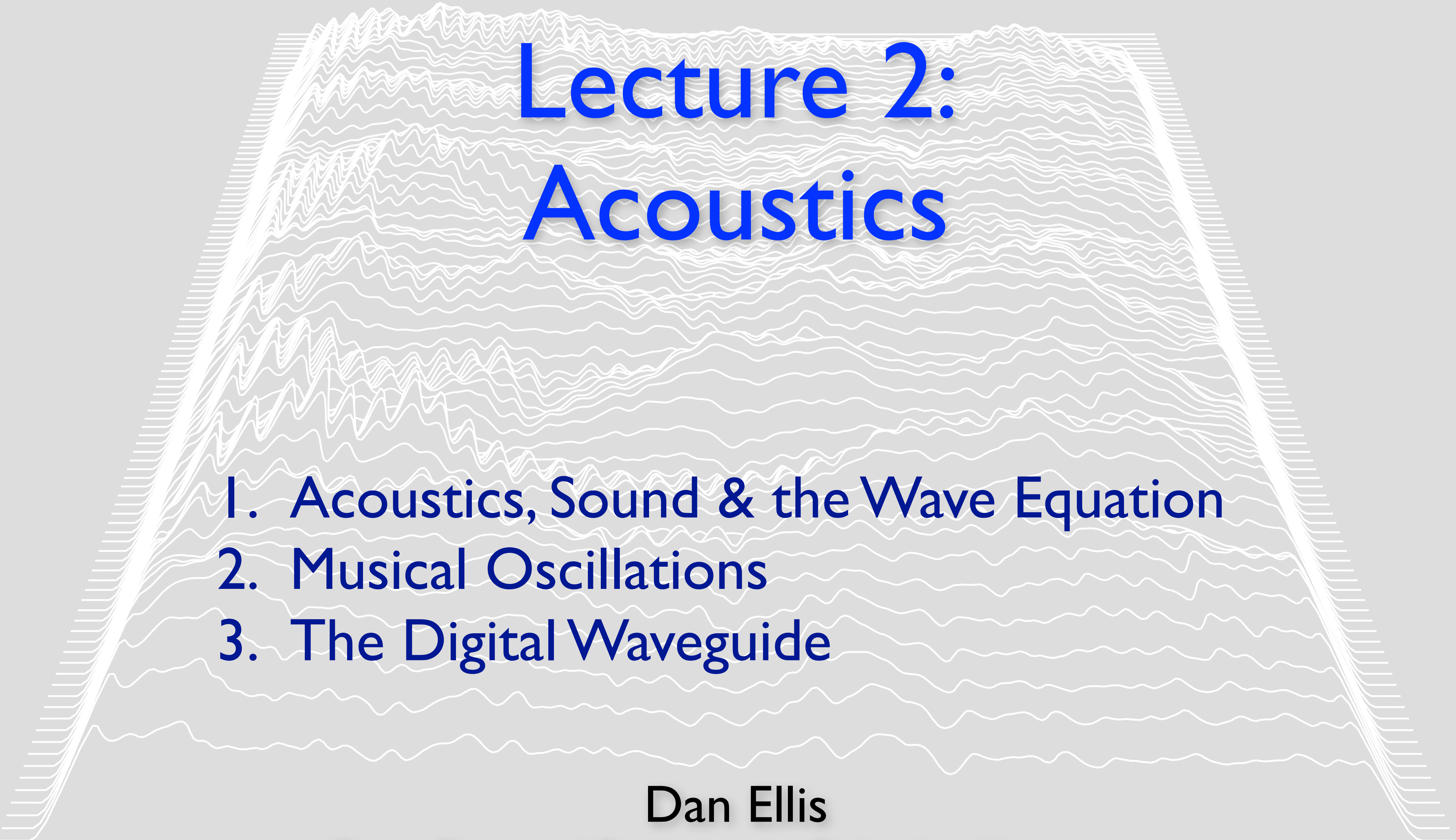
ELEN E4896 MUSIC SIGNAL PROCESSING



Lecture 2:
Acoustics

1. Acoustics, Sound & the Wave Equation
2. Musical Oscillations
3. The Digital Waveguide

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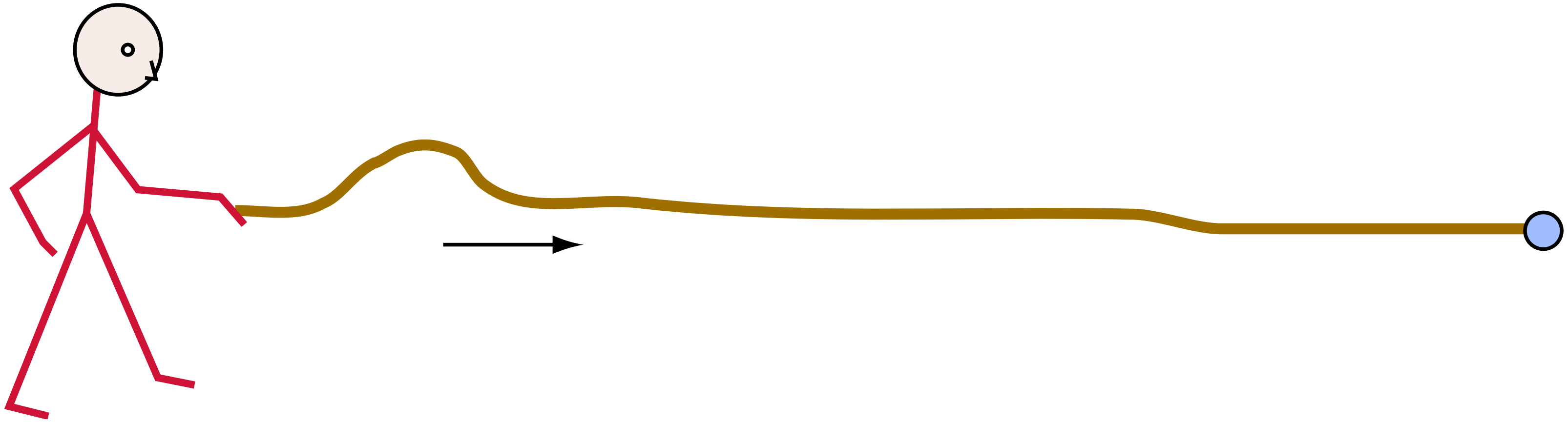
1. Acoustics & Sound

- Acoustics is the study of physical waves
- Waves transfer energy without permanent displacement of matter
- Common math for different media
    - gas, liquid, solid, EM
- Intuition: Pulse going down a rope



The Wave Equation

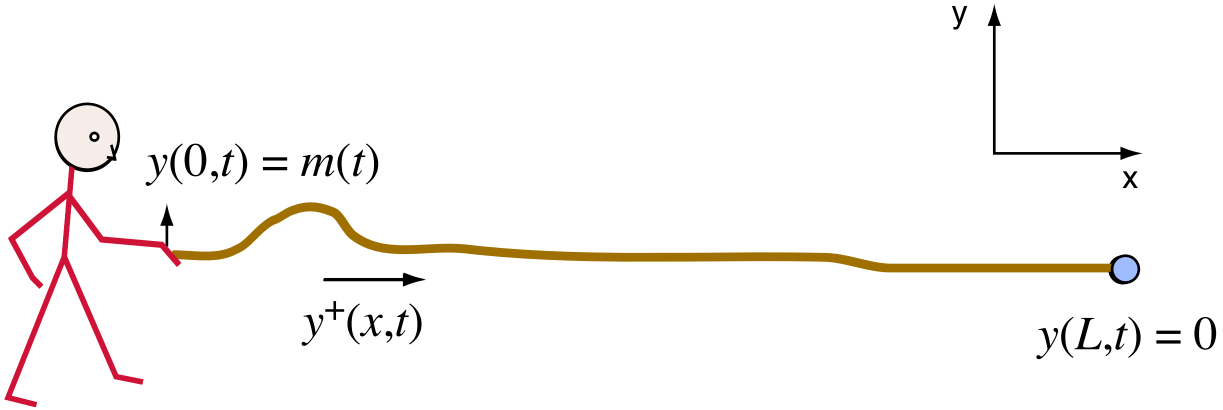
- For 1-D medium with displacement $y(x, t)$:

$$c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

curvature

acceleration

  - simple to derive from freshman physics...



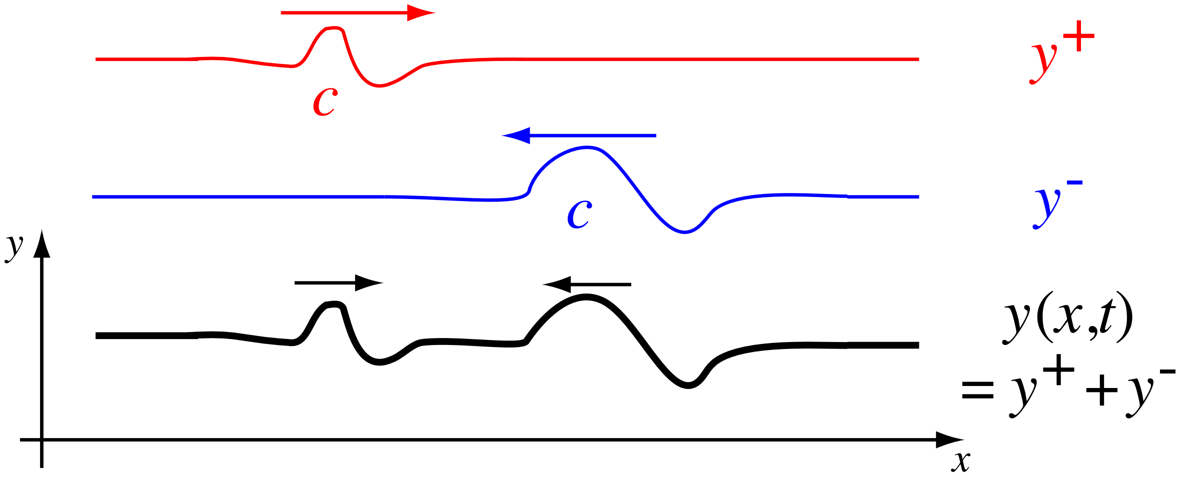
The Wave Equation

$$c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

- Solution:

$$y(x,t) = y^+(x - ct) + y^-(x + ct)$$

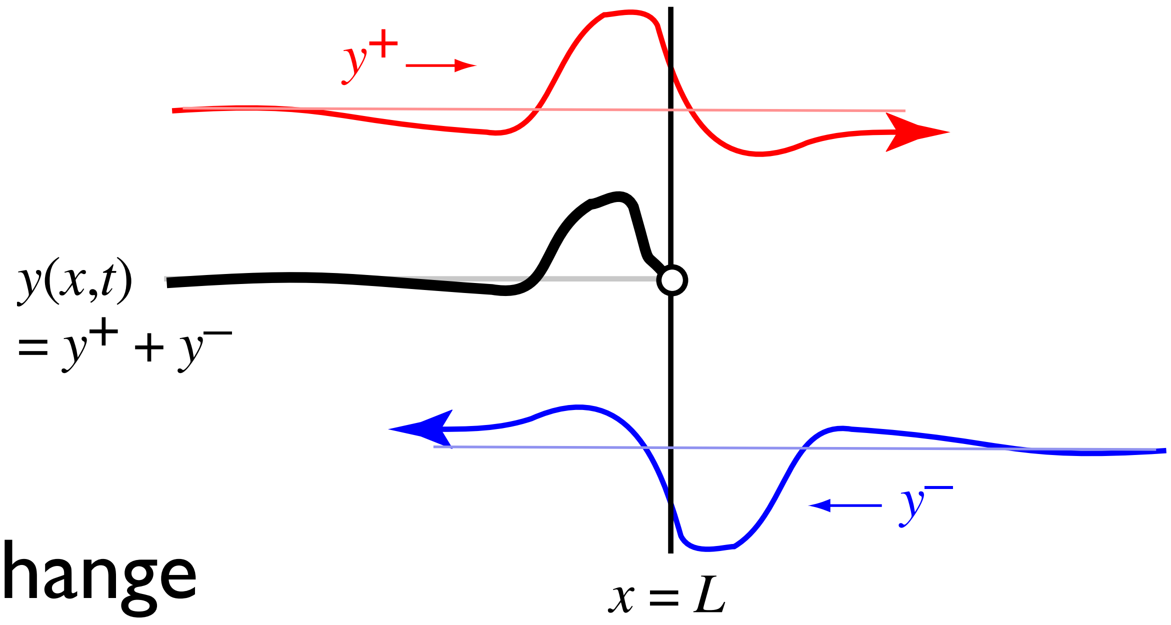
  - sum of leftward-moving y^+ and rightward-moving y^- traveling waves
  - shape does not change (set by initial conditions)



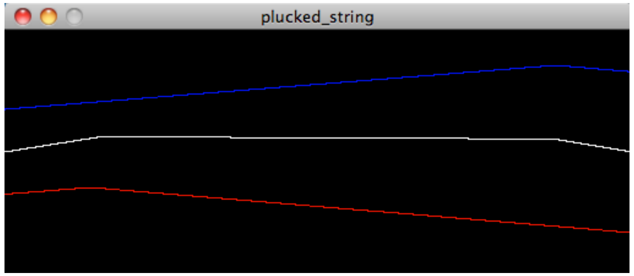
Terminations & Reflections

- Boundary conditions include fixed points
  - e.g. held ends of string

- Superposition of traveling waves must match constraints
  - hence reflections



- Any impedance change results in some reflection

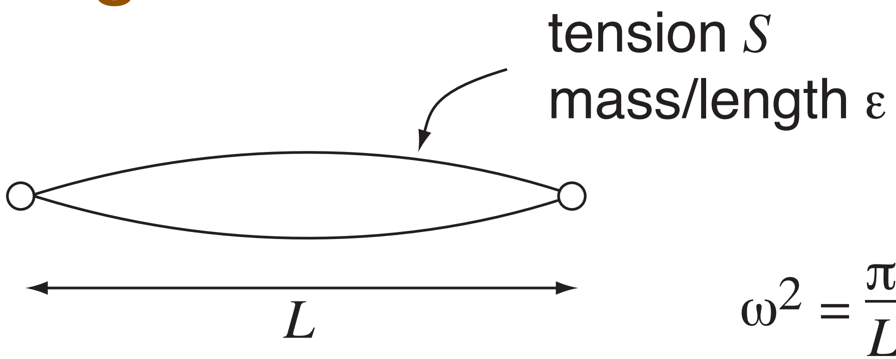


  - energy loss...

http://www.youtube.com/watch?v=YOi8suJTwx8#t=1m35s

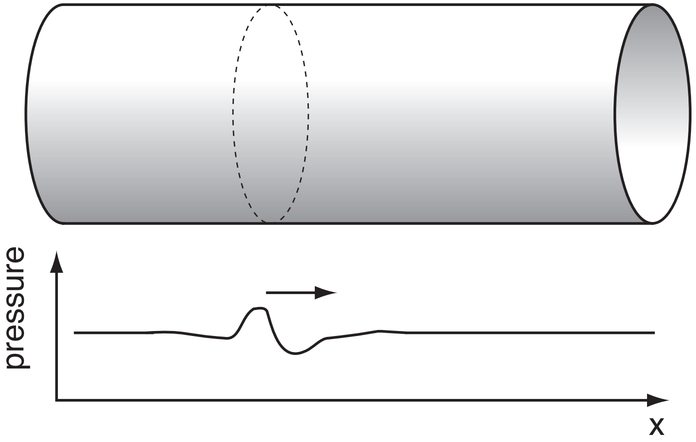
1-D Waveguides

- Plucked/struck string
  - guitar, piano ...



$$\omega^2 = \frac{\pi^2\, S}{L^2\, \varepsilon}$$

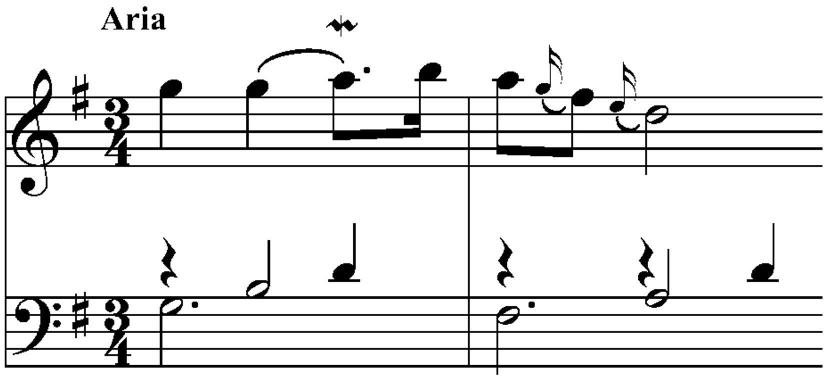
- Acoustic tube
  - e.g. clarinet or trumpet
  - vocal tract

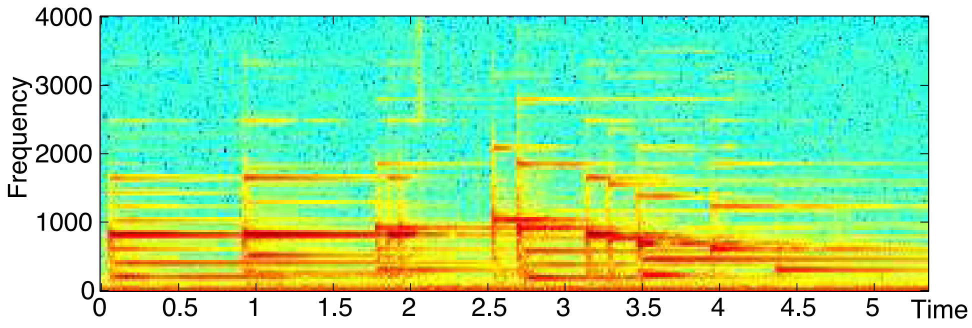


- Solid bar
  - xylophone, ...

2. Musical Oscillations

- (Pseudo) periodic oscillation is central to musical pitch





- Musical instruments create pitch in different ways...

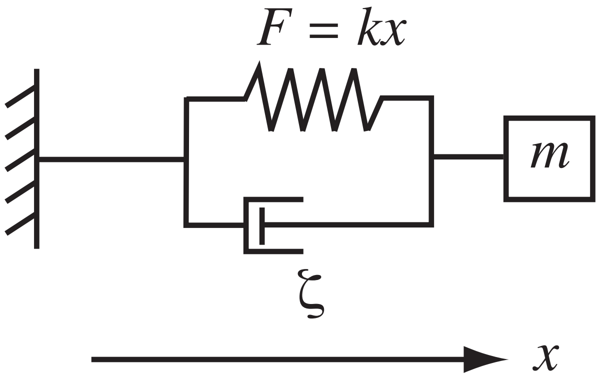
Simple Harmonic Motion

- Basic 2nd order mechanical oscillation

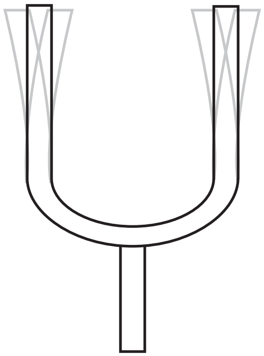
$$\ddot{x} = -\omega^2 x \qquad x = A\cos(\omega t + \phi)$$

  - resonance

- Spring + mass + damper
  - e.g. tuning fork



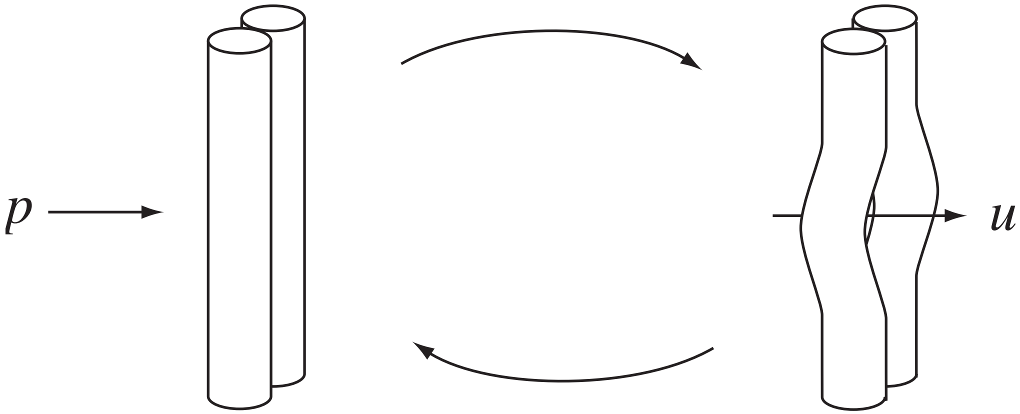
$$\omega^2 = \frac{k}{m}$$



- Not great as a musical instrument
  - only fundamental ⇒ low amplitude

Relaxation Oscillator

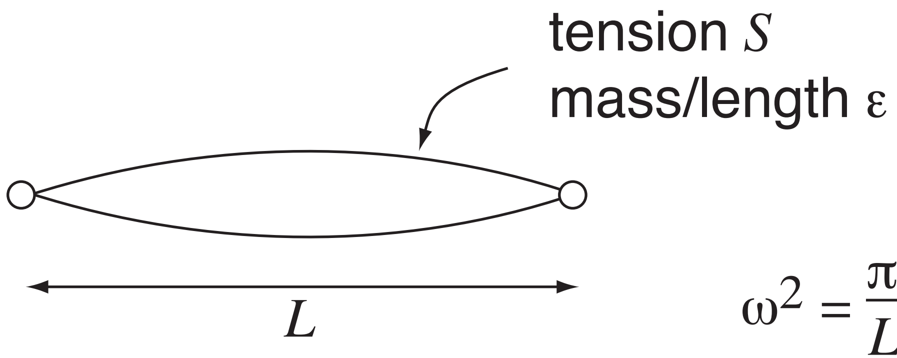
- Alternating **states**
  - 'closed state' collects energy from steady source
  - 'open state' releases energy, then reverts to closed
- e.g. Vocal folds



- Oscillation period depends on tension
  - easy to adjust
  - hard to keep stable

Strings

- Canonical wave equation example

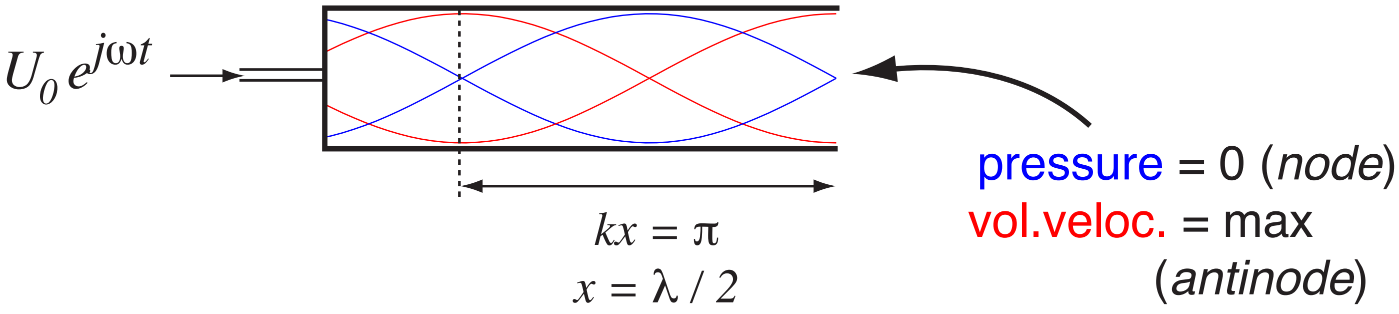


$$\omega^2 = \frac{\pi^2}{L^2}\frac{S}{\varepsilon}$$

  - guitar (plucked)
  - piano (struck)
  - violin (bowed ...)
- Control of period
  - vary length L (frets)
  - vary tension S and/or length L (piano)
- Initial conditions = excitation

Air Column

- **Wave equation in air**



$U_0 e^{j\omega t}$

$kx = \pi$

$x = \lambda / 2$

pressure = 0 (*node*)
vol.veloc. = max
(*antinode*)

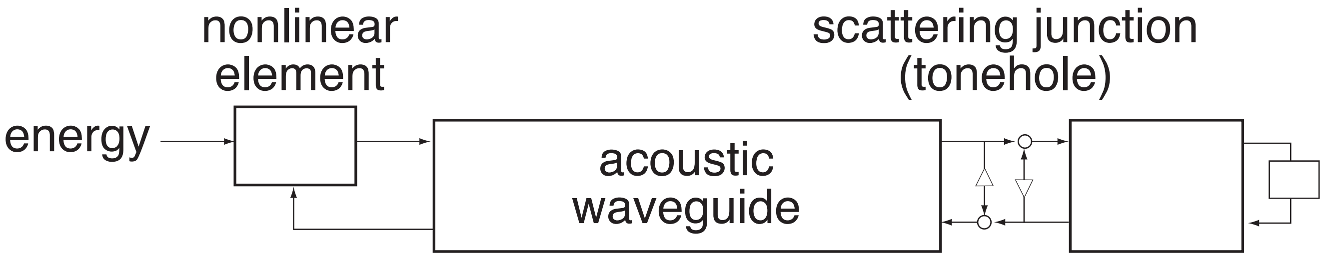
  - pressure waves traveling in tube
  - resonance of tube depends on length
  - coupled energy input

- **Clarinet, oboe, organ, flute**
  - finger holes disrupt waveguide (scattering)
  - first reflection determines oscillation period

3. Digital Waveguides

Julius Smith, 1992...

- 1-D waveguide is easily discretized
  - spatial sampling ⇔ time sampling

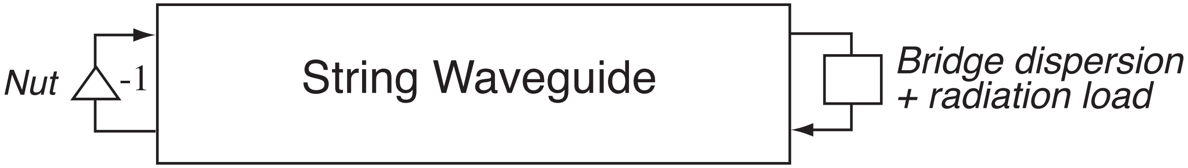


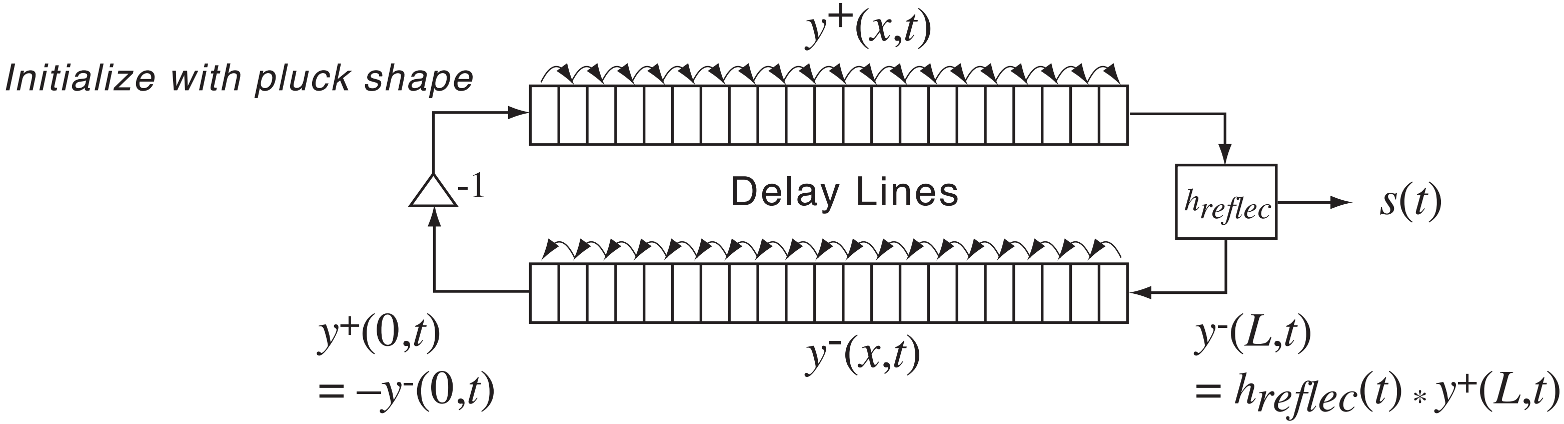
$$\omega = \frac{\pi c}{2L}$$ (quarter wavelength)

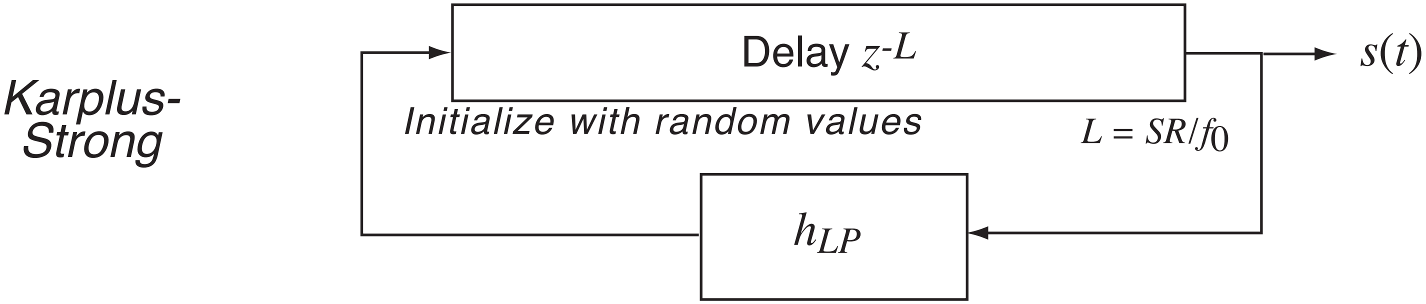
  - waveguide → delay line
  - scattering → low-pass feedback
  - final load → instrument body resonances
  - nonlinear function for energy input
  - https://ccrma.stanford.edu/~jos/wg.html

Digital Waveguides

- Direct physical model + simplifications

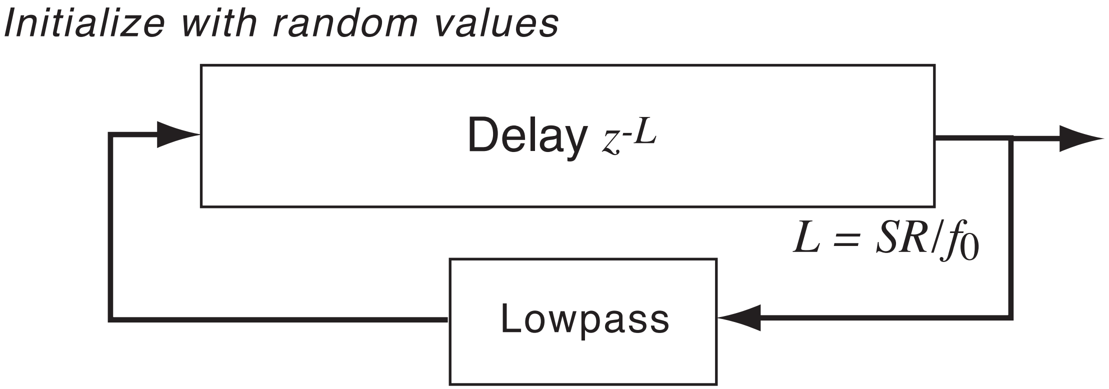




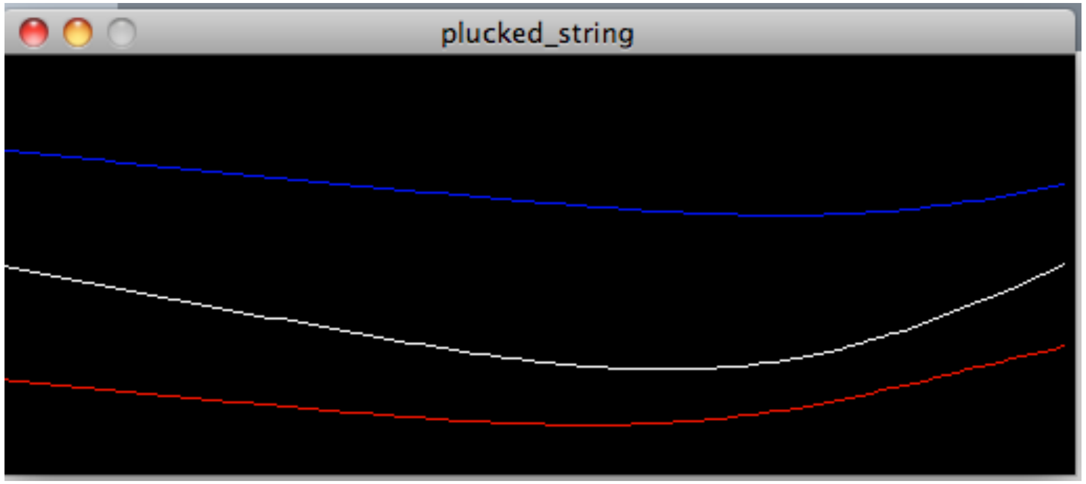


Waveguide Simulation

- Karplus-Strong Plucked String algorithm

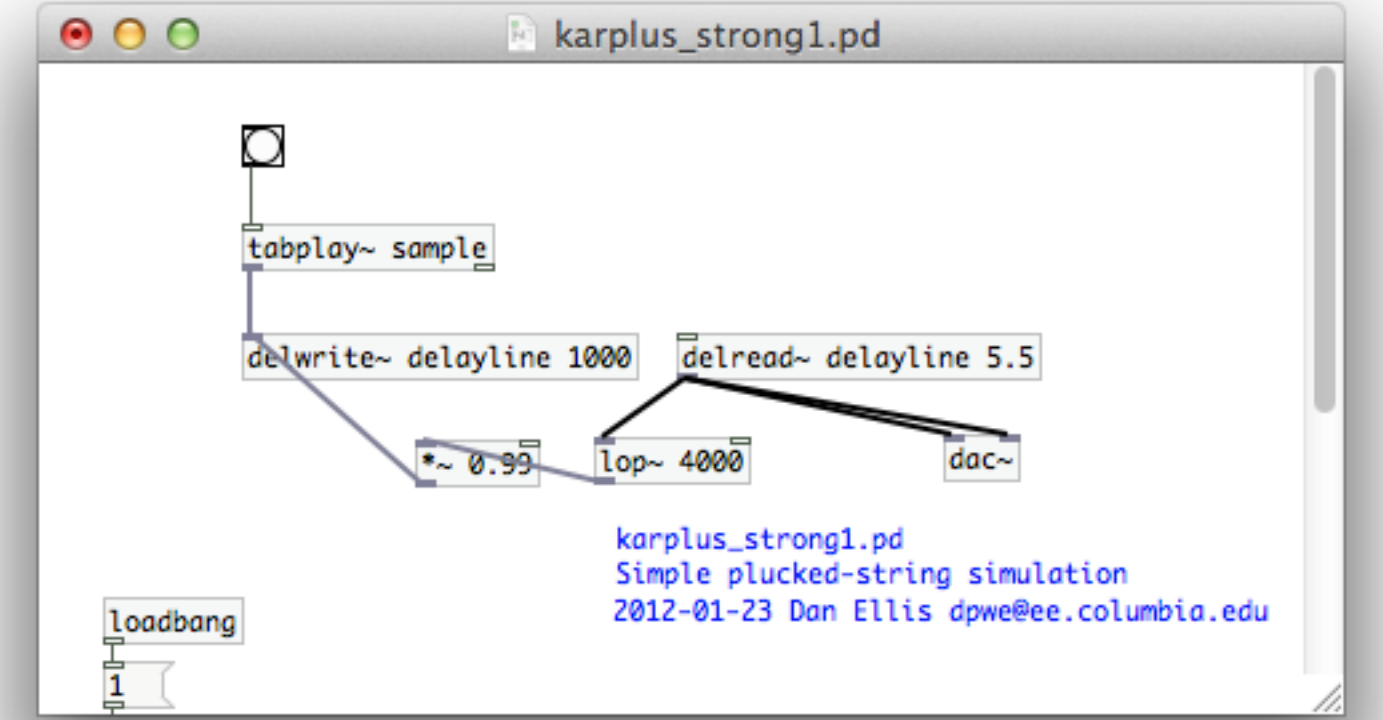


- Direct implementation of traveling waves



Karplus-Strong in Pd

- Pd’s delwrite~ / delread~ implement the delay line



karplus_strong1.pd
Simple plucked-string simulation
2012-01-23 Dan Ellis dpwe@ee.columbia.edu

http://blog.loomer.co.uk/2010/02/karplus-strong-guitar-string-synthesis.html

Summary

- Wave equation:
  Simple physics leads to oscillations

- Musical acoustics:
  Different ways to control steady tones

- Digital waveguides:
  Natural, efficient simulation of tones