4.26 Using Eq. (4.54) determine the lowest order of a lowpass elliptic filter with a 0.25-dB cutoff frequency at 1.5 kHz and a minimum attenuation of 25 dB at 6 kHz. Verify your result using `ellipord`.

4.31 A Butterworth analog highpass filter is to be designed with the following specifications: $F_p = 6.5$ kHz, $F_s = 1.5$ kHz, $\alpha_p = 0.5$ dB, and $\alpha_s = 40$ dB. What are the bandedges and the order of the corresponding analog lowpass filter? What is the order of the highpass filter? Verify your results using the function `buttord`.

M 4.4 Determine the transfer function of a lowpass elliptic analog filter with specifications as given in Problem 4.26, using Program 4_4. Plot the gain response and verify that the filter designed meets the given specifications. Show all steps.

M 4.5 Design, using MATLAB, a Butterworth analog highpass filter with specifications given in Problem 4.31. Show the transfer functions of the prototype analog lowpass and the highpass filters. Plot their gain responses and verify that both filters meet their respective specifications. Show all steps.

4.4.4 Elliptic Approximation

An *elliptic filter,* also known as a *Cauer filter,* has an equiripple passband and an equiripple stopband magnitude response, as indicated in Figure 4.21 for typical elliptic lowpass filters. The transfer function of an elliptic filter meets a given set of filter specifications, passband edge frequency Ω_p, stopband edge frequency Ω_s, passband ripple ε, and minimum stopband attenuation A, with the lowest filter order N. The theory of elliptic filter approximation is mathematically quite involved, and a detailed treatment of this topic is beyond the scope of this text. Interested readers are referred to the books by Antoniou [Ant93], Parks and Burrus [Par87], and Temes and LaPatra [Tem77].

The square-magnitude response of an elliptic lowpass filter is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 R_N^2(\Omega/\Omega_p)}, \tag{4.53}$$

where $R_N(\Omega)$ is a rational function of order N satisfying the property $R_N(1/\Omega) = 1/R_N(\Omega)$, with the roots of its numerator lying within the interval $0 < \Omega < 1$ and the roots of its denominator lying in the interval $1 < \Omega < \infty$. For most applications, the filter order meeting a given set of specifications of passband edge frequency Ω_p, passband ripple ε, stopband edge frequency Ω_s, and the minimum stopband ripple A can be estimated using the approximate formula [Ant93]

$$N \cong \frac{2\log_{10}(4/k_1)}{\log_{10}(1/\rho)}, \tag{4.54}$$

where k_1 is the discrimination parameter defined in Eq. (4.32), and ρ is computed as follows:

$$k' = \sqrt{1-k^2}, \tag{4.55a}$$

$$\rho_0 = \frac{1-\sqrt{k'}}{2(1+\sqrt{k'})}, \tag{4.55b}$$

$$\rho = \rho_0 + 2(\rho_0)^5 + 15(\rho_0)^9 + 150(\rho_0)^{13}. \tag{4.55c}$$

In Eq. (4.55a), k is the selectivity parameter defined in Eq. (4.31).