

3.19 Determine the DTFT of each of the following finite-length sequences:

$$(a) y_1[n] = \begin{cases} 1, & -N \leq n \leq N, \\ 0, & \text{otherwise,} \end{cases} \quad (b) y_2[n] = \begin{cases} 1, & 0 \leq n \leq N, \\ 0, & \text{otherwise,} \end{cases} \quad (c) y_3[n] = \begin{cases} 1 - \frac{|n|}{N}, & -N \leq n \leq N, \\ 0, & \text{otherwise,} \end{cases}$$

$$(d) y_4[n] = \begin{cases} N + 1 - |n|, & -N \leq n \leq N, \\ 0, & \text{otherwise,} \end{cases} \quad (e) y_f[n] = \begin{cases} \cos(\pi n/2N), & -N \leq n \leq N, \\ 0, & \text{otherwise.} \end{cases}$$

3.24 Prove the following theorems of the discrete-time Fourier transform: (a) Linearity theorem, (b) Time-reversal theorem, (c) Time-shifting theorem, and (d) Frequency-shifting theorem.

3.30 The magnitude function $|X(e^{j\omega})|$ of a discrete-time sequence $x[n]$ is shown in Figure P3.1 for a portion of the angular frequency axis. Sketch the magnitude function for the frequency range $-\pi \leq \omega < \pi$. What type of sequence is $x[n]$?

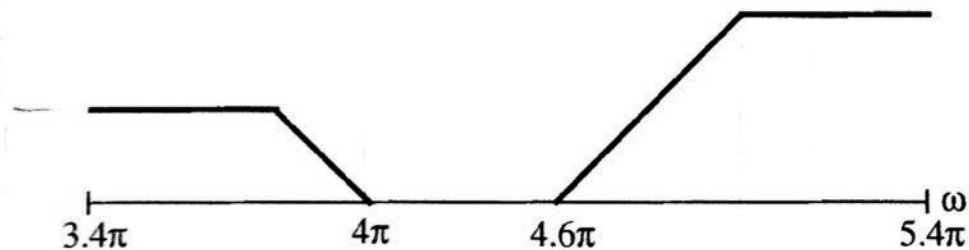


Figure P3.1

5.8 Determine the N -point DFTs of the following length- N sequences defined for $0 \leq n \leq N - 1$:

$$(a) x_a[n] = \sin(2\pi n/N), \quad (b) x_b[n] = \cos^2(2\pi n/N), \quad (c) x_c[n] = \cos^3(2\pi n/N).$$

5.15 Let $x[n]$, $0 \leq n \leq N - 1$, be a length- N sequence with an N -point DFT given by $X[k]$, $0 \leq k \leq N - 1$. Determine the $2N$ -point DFT of each of the following length- $2N$ sequences:

$$(a) g[n] = \begin{cases} x[n], & 0 \leq n \leq N - 1, \\ 0, & N \leq n \leq 2N - 1, \end{cases} \quad (b) h[n] = \begin{cases} 0, & 0 \leq n \leq N - 1, \\ x[n], & N \leq n \leq 2N - 1. \end{cases}$$

x[n-N] to make sense

M 3.2 Using Program 3_1, determine and plot the real and imaginary parts and the magnitude and phase spectra of the DTFTs of the sequences of Problem 3.19 for $N = 10$.

M 5.2 Write a MATLAB program to compute the circular convolution of two length- N sequences via the DFT-based approach. Using this program, determine the circular convolution of the following pairs of sequences:

$$(a) g[n] = \{5, -2, 2, 0, 4, 3\}, \quad h[n] = \{3, 1, -2, 2, -4, 4\},$$

$$(b) x[n] = \{2 - j, -1 - j3, 4 - j3, 1 + j2, 3 + j2\}, \quad v[n] = \{-3, 2 + j4, -1 + j4, 4 + j2, -3 + j\},$$

$$(c) x[n] = \cos(\pi n/2), \quad y[n] = 3^n, \quad 0 \leq n \leq 4.$$

Verify your result using the function `circonv`.

M 5.3 Using MATLAB, verify the symmetry relations of the DFT of a complex sequence as listed in Table 5.1.

Table 5.1: Symmetry properties of the DFT of a complex sequence.

| Length- N Sequence | N -point DFT |
|---|--|
| $x[n] = x_{\text{re}}[n] + jx_{\text{im}}[n]$ | $X[k] = X_{\text{re}}[k] + jX_{\text{im}}[k]$ |
| $x^*[n]$ | $X^*[\langle -k \rangle_N]$ |
| $x^*[\langle -n \rangle_N]$ | $X^*[k]$ |
| $x_{\text{re}}[n]$ | $X_{\text{cs}}[k] = \frac{1}{2}\{X[k] + X^*[\langle -k \rangle_N]\}$ |
| jx_{im} | $X_{\text{ca}}[k] = \frac{1}{2}\{X[k] - X^*[\langle -k \rangle_N]\}$ |
| $x_{\text{cs}}[n]$ | $X_{\text{re}}[k]$ |
| $x_{\text{ca}}[n]$ | $jX_{\text{im}}[k]$ |

Note: $x_{\text{cs}}[n]$ and $x_{\text{ca}}[n]$ are the circular conjugate-symmetric and circular conjugate-antisymmetric parts of $x[n]$, respectively. Likewise, $X_{\text{cs}}[k]$ and $X_{\text{ca}}[k]$ are the circular conjugate-symmetric and circular conjugate-antisymmetric parts of $X[k]$, respectively.

3.19 Determine the DTFT of each of the following finite-length sequences:

$$\begin{aligned}
 \text{(a) } y_1[n] &= \begin{cases} 1, & -N \leq n \leq N, \\ 0, & \text{otherwise,} \end{cases} &
 \text{(b) } y_2[n] &= \begin{cases} 1, & 0 \leq n \leq N, \\ 0, & \text{otherwise,} \end{cases} &
 \text{(c) } y_3[n] &= \begin{cases} 1 - \frac{|n|}{N}, & -N \leq n \leq N, \\ 0, & \text{otherwise,} \end{cases} \\
 \text{(d) } y_4[n] &= \begin{cases} N + 1 - |n|, & -N \leq n \leq N, \\ 0, & \text{otherwise,} \end{cases} &
 \text{(e) } y_f[n] &= \begin{cases} \cos(\pi n/2N), & -N \leq n \leq N, \\ 0, & \text{otherwise.} \end{cases}
 \end{aligned}$$