Towards a model for discrimination of broadband signals

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The conventional model for broadband discrimination assumes that resolution is limited by peripheral internal noise that is statistically independent across channels. In this paper, we extend this model in a number of directions. In particular, we compute, compare, and discuss the effects of interchannel correlation and central noise on the sensitivity index d', for discrimination of overall level and discrimination of spectral shape.

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INTRODUCTION

A major thrust in current hearing research is the study of psychophysical and physiological responses to broadband stimuli. Such study is important for applications because most natural stimuli are broadband. Also, it appears particularly appropriate at this time from the viewpoint of auditory science. Although responses to narrowband stimuli are not yet fully understood, sufficient progress has been made to suggest that systematic research on broadband stimuli can be highly productive.

In this paper, we consider issues, make comments, and ask questions related to the way in which the auditory system processes (i.e., compares or combines) information in different frequency channels when faced with discrimination tasks involving broadband (but stationary) stimuli. We assume throughout that the listener's task is to discriminate between two stimuli S_1 and S_2 using a one-interval, two-alternative, forced-choice discrimination paradigm. The focus is on auditory operations that occur central to the peripheral frequency analysis, on ideal processing, and on internal-noise structure. Also, the models that are presented are meant to be illustrative, not definitive.

In Sec. I, we briefly review a simple model based on peripheral frequency analysis, internal noise that is statistically independent across frequency channels, and ideal central processing. The sensitivity index d' in this case is given simply by the square root of the sum of the squares of the sensitivity indices for the individual channels. In Sec. II, we relax the assumption that the noise is independent across channels and examine sensitivity d' for a noise structure that incorporates a particularly simple form of interchannel correlation, namely, one in which the interchannel correlation is the same for all pairs of channels. This correlation can be regarded as the result of an internal noise component that is common to all channels or as the result of an external noise associated with the stimulus presentation. One case in the latter category that is of particular interest to us in this paper is that in which the experimenter randomly roves the stimulus level in order to eliminate level cues and force the listener to discriminate solely on the basis of spectral shape. In addition to computing d' for this correlated noise structure, we consider a transformation that replaces the correlated channel variables by a new set of variables that are (1) statistically independent and (2) naturally suited to comparing level discrimination with spectral-shape discrimination. In Sec. III, we examine the dependence of d' on stimulus parameters and noise parameters for both level discrimination and spectral-shape discrimination. In Sec. IV, we generalize the results of Sec. III by including central noise in addition to the peripheral noise. Two types of central noise are considered: noise that is added after the decision variable is formed and noise that is added before the decision variable is formed but after the above-mentioned transformation of variables is effected. These two types of central noise lead to quite different results. For example, if one attempts to estimate the ratio of the correlated to uncorrelated noise in the periphery without taking account of central noise, the first leads to an overestimate of the ratio whereas the latter leads to an underestimate. In Sec. V, we make some concluding remarks and comment on future research.

In general, the issues considered in this paper are independent of whether the variables in question are monaural or binaural, whether they are derived from amplitude measures or phase measures, and whether the amplitude measures are derived from neural firing rate or neural firing synchrony. They are even independent of the type of channels involved (e.g., the channels could refer to time intervals rather than frequency intervals). Nevertheless, since we are mainly concerned here with the processing of information across frequency channels, we shall not hesitate to use language (such as the word "spectrum") normally associated with the frequency variable.

I. INDEPENDENT PERIPHERAL NOISE

The independent-noise model, illustrated schematically in Fig. 1, has been used previously by a number of investigators (e.g., Plomp, 1970; Florentine and Buus, 1981). It is usually assumed in this model that (1) there is a linear filter bank which resolves the input signal into N frequency bands; (2) the internal noise in the processing can be represented by Gaussian noise that is statically independent across frequency channels; (3) the variance of the noise in each channel is independent of which stimulus was presented; and (4) the central processing is ideal within the constraints imposed by the internal noise. Under these conditions, the "internal spectrum" generated by the stimulus presentation can be

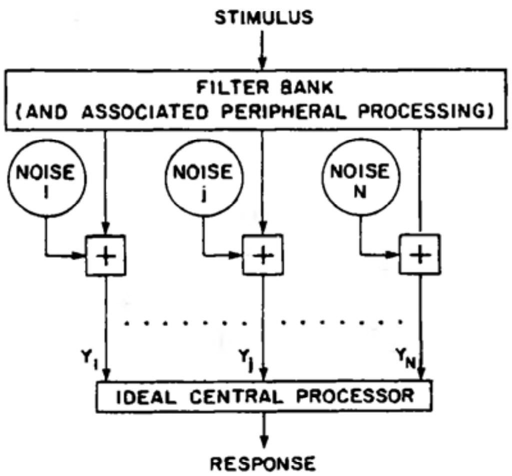

FIG. 1. Independent noise. In this model, the Y_j are statistically independent Gaussian random variables with means M_{ij} and variances σ_j^2.

represented by an N-dimensional random vector $[Y_1,...,Y_N]$ where the Y_j are statistically independent Gaussian random variables with means $M(Y_j|S_i) = M_{ij}$ and variances $V(Y_j|S_i) = \sigma_j^2$ independent of i $(1 \leqslant j \leqslant N, 1 \leqslant i \leqslant 2)$. The decision variable λ for the ideal central processor can be identified with the logarithm of the likelihood ratio

$$\lambda = \log \frac{P(Y_1,...,Y_N|S_1)}{P(Y_1,...,Y_N|S_2)} \tag{1}$$

or with any monotonic transformation of it. The sensitivity index d' for this model is given by

$$d' = \left[\sum_{j=1}^{N} \left(\frac{\Delta_j}{\sigma_j}\right)^2\right]^{1/2}, \quad \Delta_j = M_{1j} - M_{2j}. \tag{2}$$

The values of the quantities Δ_j/σ_j (which are the sensitivity indices for the individual channels) depend both on the discrimination task (i.e., on the choice of S_1 and S_2) and on the details of the model.

In a frequently used version of the independent-noise model, S_1 and S_2 are monaural stimuli that differ solely in their power spectra and it is assumed that each filter is followed by a "level estimator" which estimates the filter output level. The level estimate is assumed to involve rectification, integration, and a logarithmic transformation; and the internal noise is added after the level estimate is formed. Since the noise occurs after the logarithmic transformation, the model implies that Weber's law is valid for intensity discrimination. Since the noise occurs after the integration, the model implies that intensity just-noticeable differences (jnds) are independent of duration. Although neither of these implications are entirely consistent with the data, they do provide crude first-order approximations.[1] In this version of the above model, the means M_{ij} represent mean levels (in dB) and the σ_j represent "Weber-law noise." If absolute-threshold phenomena are of interest, then other noise must be added (before the logarithmic transformation). If S_1 and S_2 are probabilistic rather than deterministic, then this stimulus variability ("external noise") must also be taken into account in the formation of the variance term. In principle, this model is applicable to all experiments in which the differences beween S_1 and S_2 can be described in terms of differences in their power spectra, including experiments on detection, intensity and frequency jnds, and broadband spectral-shape (i.e., timbre) jnds. It can also be shown that under certain conditions it is applicable to experiments on spectral shape discrimination even when the roving-level technique is used to eliminate cues based on overall level changes or level changes within a single band (and thereby to force the listener to compare levels in different frequency bands). Although the presence of a random roving level introduces correlation among the output levels in the different frequency bands, the effect of this correlation is eliminated when the central processor compares levels in the different frequency bands. This issue is discussed further in Sec. II.

Other versions of the independent-noise model can be realized in the domain of binaural hearing. Suppose, for example, that S_1 and S_2 are binaural stimuli, $S_1 = (S_{1L}, S_{1R})$ and $S_2 = (S_{2L}, S_{2R})$, and that S_1 and S_2 differ only with respect to their interaural phase spectra, $\phi(S_{1L}) - \phi(S_{1R})$ and $\phi(S_{2L}) - \phi(S_{2R})$. Assume, furthermore, that corresponding filters in the two ears are followed by a device that estimates interaural phase and that the internal noise is added to this estimate. The comments stated above for discrimination of monaural power spectra then have direct analogs for the discrimination of interaural phase spectra. In particular, the model can be applied to experiments on the discrimination of the shape of the interaural phase spectrum which employ the roving phase technique to eliminate cues based on overall interaural phase changes or interaural phase changes within a single band and thereby to force the listener to compare interaural phases across frequency bands. The case of interaural amplitude spectra is analogous to that of interaural phase spectra with the quantity $\phi(S_{iL}) - \phi(S_{iR})$ replaced by log amp(S_{iL}) − log amp(S_{iR}), and roving interaural phase replaced by roving interaural amplitude.

II. INTERCHANNEL CORRELATION

We now generalize the independent-noise model by adding the common noise variable R to the variables Y_j to form the correlated variables $Z_j = Y_j + R$ (see Fig. 2). We assume that R is a statistically independent Gaussian random variable with mean zero and variance σ_R^2. (When $\sigma_R = 0$, the model obviously reduces to the independent-noise model.)

The random variable R, which introduces a particulary simple form of interchannel correlation, can be interpreted in two ways. First, it can be viewed as the result of a random rove introdued by the experimenter to force the listener to make interchannel comparisons (i.e., to attend to spectral *shape*) rather than respond on the basis of single-channel information. This interpretation is applicable, for example, to the experiments performed by Green and his associates (e.g., Spiegel and Green, 1982; Spiegel *et al.*, 1981; Green *et al.*, 1983; Mason *et al.*, 1984) and in our laboratory (e.g., Farrar *et al.*, 1986) on the discrimination of the shape of the power spectrum in which the overall level is roved.[2] It is also applicable to experiments on the discrimination of the shape of the interaural phase spectrum in which the overall inter-

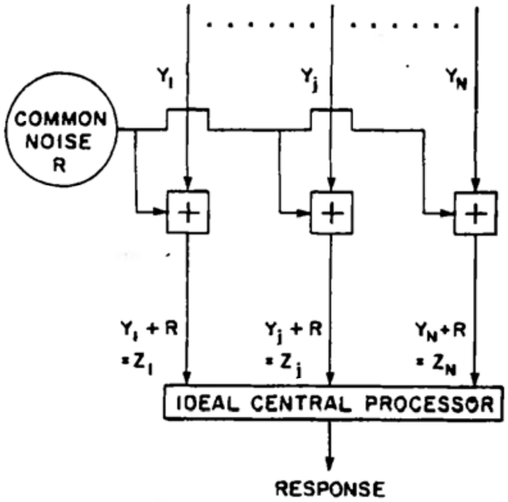

FIG. 2. Interchannel correlation. In this model, the Y_j of the model pictured in Fig. 1 are modified by adding the statistically independent random variable R. The results would be identical if instead of adding the common noise R after the independent noises pictured in Fig. 1, it was added before the independent noises (as would be appropriate when R represents a roving level in the stimulus introduced by the experimenter).

aural phase is roved. Second, the random variable R can be viewed as representing internally generated peripheral noise that is common to all channels (as might be caused, for example, by respiratory or circulatory processes). Although a realistic representation of interchannel correlation for internally generated noise would obviously require a more complex structure (e.g., a reduction in correlation as the channels become more separated in frequency), the above representation provides greatly simplified mathematics and important insight.

With this model, the internal spectrum is given by $[Z_1,...,Z_N]$, where the Z_j are Gaussian random variables with means M_{ij}, variances $\sigma_j^2 + \sigma_R^2$, and covariances σ_R^2. Sensitivity d' for this case (see Appendix A) is given by

$$d' = \left[\sum_{j=1}^{N}\left(\frac{\Delta_j}{\sigma_j}\right)^2 - E\right]^{1/2}, \tag{3}$$

where

$$E = \sigma_R^2\left(1 + \sigma_R^2 \sum_{j=1}^{N}\frac{1}{\sigma_j^2}\right)^{-1}\left(\sum_{j=1}^{N}\frac{\Delta_j}{\sigma_j^2}\right)^2. \tag{4}$$

Equation (3) can be approximated by Eq. (2) to the extent that E is small relative to $\Sigma(\Delta_j/\sigma_j)^2$. Two situations in which this is the case are:

(1) the discrimination experiment does not make use of the roving procedure and the internal channels are uncorrelated (i.e., $\sigma_R = 0$),

(2) the stimuli S_1 and S_2 are "balanced" [i.e., $\Sigma_{j=1}^N(\Delta_j/\sigma_j^2) = 0$].

In the second situation, although the roving procedure may be used or the internal noise may be correlated across channels, there is no effect on sensitivity.

In the remainder of this paper, it will be assumed for simplicity that

$$\sigma_j = \sigma, \quad 1 \leqslant j \leqslant N. \tag{5}$$

With this assumption, Eq. (3) becomes

$$d' = \frac{1}{\sigma}\left[\sum_{j=1}^{N}\Delta_j^2 - \frac{\sigma_R^2}{\sigma^2 + N\sigma_R^2}\left(\sum_{j=1}^{N}\Delta_j\right)^2\right]^{1/2}. \tag{6}$$

This equation reduces to Eq. (2) if either there is no correlated component ($\sigma_R = 0$) or S_1 and S_2 are balanced ($\Sigma\Delta_j = 0$). Note also that this equation can be rewritten in the form

$$d' = \frac{1}{\sigma}\left(\sum_{j=1}^{N}(\Delta_j - \overline{\Delta})^2 + \frac{N\sigma^2}{\sigma^2 + N\sigma_R^2}\overline{\Delta}^2\right)^{1/2}, \tag{7}$$

where $\overline{\Delta} = (1/N)\Sigma_{j=1}^N\Delta_j$ is the mean value of Δ_j. The second term in Eq. (7) is the contribution of d' arising from the mean difference between S_1 and S_2 across channels (i.e., the difference in average level), and the first is the contribution that arises from differences that remain after normalization for average level (i.e., the differences in spectral shape).

Further insight into the ideal processor can be obtained by transforming the random variables Z_j into new random variables X_j, where

$$X_1 = \sum_{j=1}^{N} Z_j, \tag{8}$$

$$X_j = \sqrt{\frac{j-1}{j}}\left(Z_j - \frac{1}{j-1}\sum_{k=1}^{j-1} Z_k\right), \quad 2 \leqslant j \leqslant N. \tag{9}$$

The variable X_1 is proportional to the mean level (i.e., Z_j averaged over channels), whereas X_j ($j \geqslant 2$) is proportional to the difference between Z_j and the mean of the first $j-1$ levels. The d' obtained by ideal processing of the X_j is identical to the d' obtained by ideal processing of the Z_j since the transformation can be inverted. Moreover, the X_j are statistically independent (see Appendix B). The means and variances of these variables are given by (see Appendix C)

$$M(X_1|S_i) = \sum_{j=1}^{N} M_{ij}, \quad V(X_1) = N(\sigma^2 + N\sigma_R^2) \tag{10}$$

$$M(X_j|S_i) = \sqrt{\frac{j-1}{j}}\left(M_{ij} - \frac{1}{j-1}\sum_{k=1}^{j-1} M_{ik}\right),$$

$$V(X_j) = \sigma^2, \quad 2 \leqslant j \leqslant N. \tag{11}$$

If S_1 and S_2 are balanced (i.e., $\Sigma M_{1j} = \Sigma M_{2j}$), then $P(X_1|S_1) = P(X_1|S_2)$ and the variable X_1 can be ignored (since it carries no discrimination information). Note that once X_1 is eliminated, the set of variables consists solely of weighted interchannel differences and the parameter R is cancelled out. Note, also that ideal processing of these statistically independent weighted-difference variables produces the same results as ideal processing of the *correlated* adjacent differences $D_j = Z_j - Z_{j-1}$ (since the transformation from $X_2,...,X_N$ to $D_2,...,D_N$ can be inverted).

If the difference between the stimuli S_1 and S_2 is constant across channels (i.e., $\Delta_j = \Delta$ independent of j), then $P(X_j|S_1) = P(X_j|S_2)$, $2 \leqslant j \leqslant N$, and all the variables *except* X_1 can be ignored. Thus the vector $[X_1,...,X_N]$ provides the observer with a data display that is particularly well-suited for both the discrimination of level and the discrimination of spectral shape. For the former tasks, all the relevant information is in X_1, whereas for the latter tasks, it is all in $[X_2,...,X_N]$.

Finally, it should be realized that the transformation specified by Eqs. (8) and (9) is not unique: other transformations exist which provide statistically independent variables that can be subdivided into a mean-level variable and level-difference variables.[3] Furthermore, the equivalence of all these sets of variables with respect to predicted sensitivity is valid only to the extent that all further processing is ideal and, in particular, that the remainder of the processing is noise-free. If the subsequent processing introduces additional noise, the different sets of variables may not be equivalent.

III. LEVEL DISCRIMINATION VERSUS SPECTRAL-SHAPE DISCRIMINATION

In an experiment on the discrimination of *level*, S_1 and S_2 will not be balanced so that a nonzero value of σ_R will reduce performance. This will occur independent of whether σ_R results from an experimentally introduced roving level or from correlated internal noise. Assuming, for simplicity, that $\Delta_j = \Delta$ independent of j (i.e., S_1 and S_2 differ by a constant increment across channels), we find [from Eq. (6)] that d' reduces to

$$d'_L = \sqrt{N}\,\Delta/\sqrt{\sigma^2 + N\sigma_R^2}, \tag{12}$$

where d'_L denotes the sensitivity index for level discrimination. For $\sigma^2 \gg N\sigma_R^2$, one has $d'_L = \sqrt{N}\,\Delta/\sigma$, the usual independent-noise result; for $\sigma^2 \ll N\sigma_R^2$, one has $d'_L = \Delta/\sigma_R$ and d'_L is seen to be independent of N as well as σ. Letting $\Delta_N^{(0)}$ denote the level jnd for the case of N bands, we find that the dependence of the jnd on N is described by the equation

$$\frac{\Delta_N^{(0)}}{\Delta_1^{(0)}} = \sqrt{\frac{1 + \sigma^2/N\sigma_R^2}{1 + \sigma^2/\sigma_R^2}}\,. \tag{13}$$

Note that even a modest amount of correlation substantially reduces the $\sqrt{N}$ dependence. For example, if $\sigma_R^2 = \sigma^2/3$, the ratio $\Delta_N^{(0)}/\Delta_1^{(0)}$ never decreases below 0.5 (even for $N = \infty$). Note also the implications of this correlation for the "near miss" to Weber's law for the intensity discrimination of narrowband signals: the correlation reduces the effect of "spread of excitation" to neighboring frequency channels as intensity is increased (e.g., see Florentine and Buus, 1981).

In contrast to discrimination of level, in a typical experiment on the discrimination of spectral *shape*, S_1 and S_2 are balanced (at least approximately) so that there are no overall level cues. In this case, the results are independent of the value of σ_R and are the same as those obtained when $\sigma_R = 0$. In other words, sensitivity in discrimination of spectral shape is unaffected by correlation among frequency channels, independent of whether this correlation arises from an experimentally introduced roving level or from a common internal noise component. The lack of dependence on roving level is consistent, at least for the special case of power spectra, with experimental data (e.g., Mason *et al.*, 1984; Farrar *et al.*, 1986).

Assuming that S_1 and S_2 are balanced ($\Sigma\Delta_j = 0$) and furthermore that the level increments across channels differ only in sign (i.e., $\Delta_j = \pm\Delta$), one obtains [from Eq. (6)] the simple expression

$$d'_S = \sqrt{N}\,(\Delta/\sigma), \tag{14}$$

where d'_S denotes the sensitivity index for shape discrimination.

Consider now a pair of experiments in which the parameters are chosen to satisfy the assumptions of Eqs. (12) and (14) and in which the value of Δ in the level-discrimination experiment is the same as in the shape-discrimination experiment. Under these conditions, we have

$$(d'_S/d'_L)^2 = 1 + N(\sigma_R^2/\sigma^2). \tag{15}$$

This result, at least in principle, suggests a method for evaluating the ratio of the correlated and uncorrelated noise components.[4]

IV. CENTRAL NOISE

In addition to the noise components already considered, all of which reflect variability in the stimulus or in the peripheral processing, it is necessary to consider noise components that reflect variability in the central processing ("central noise"). Although it is often difficult to separate the effects of central noise from those of peripheral noise, such separation is important both for the development of improved black-box models and for efforts to explain perceptual resolution in terms of the variability evident in physiological responses (e.g., Siebert, 1968, 1973; Colburn, 1973, 1984, 1986; Luce and Green, 1972; Teich and Lachs, 1979; Lachs *et al.*, 1984).

Central noise is likely to arise from a variety of sources. For example, it will be generated by imperfections in the ability to store stimulus sensations for use later in time ("memory noise"). Efforts to incorporate memory noise into a model of discrimination and identification for unidimensional stimulus sets have been reported in our series of papers on intensity perception (e.g., Durlach and Braida, 1969; Braida *et al.*, 1984).

Central noise will also be generated by imperfections in the ability to set and maintain constant decision criteria ("criterion noise"). Recently, Treisman and his colleagues have used sequential effects to estimate lower bounds on the ratio of criterion noise to sensory noise (e.g., Treisman and Williams, 1984).

Particularly relevant to the primary concerns of this paper, central noise is also likely to arise in connection with attempts to compare channel levels across frequency ("frequency-comparison noise"). Intuitively, it seems obvious that this noise will increase as the frequency bands being compared are separated further in frequency. This intuitive notion is strongly supported by the data of Lim *et al.* (1977) which show how intensity resolution for tones degrades as the tones to be compared are separated in frequency. According to these data, resolution degrades by roughly a factor of 3 as the two frequencies used in the level comparison vary from (1000 Hz, 1000 Hz) to (1000 Hz, 650 Hz). The notion is *not* supported, however, by the data from Green and his associates. These data (e.g., Fig. 2 in Green, 1983)

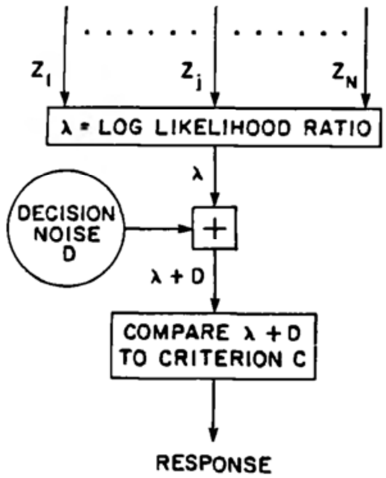

FIG. 3. Central noise case 1. In this model, the Z_j of the model pictured in Fig. 2 are processed by a degraded (nonideal) central processor. The degradation in this case consists of adding independent Gaussian noise to the decision variable λ. The same results would be obtained if the noise were added to the decision criterion C.

suggest that there is no significant degradation with frequency separation.

An important difference bertween these two experiments is that in the former experiment the stimuli to be compared were presented sequentially whereas in the latter experiment they were presented simultaneously. Thus the two experiments differ with respect to both masking and memory factors. Preliminary analysis suggests that the difference between the two sets of empirical results cannot be explained solely in terms of masking phenomena. On the other hand, the notion that a significant portion of the degradation with frequency separation observed with the sequential presentation lies in the memory domain rather than the stimulus-sensation domain makes little sense intuitively. Clearly, further experiments are needed to clarify this issue.

Another point directly relevant to the present paper concerns the way in which various forms of central noise appear as peripheral noise when theoretical estimates are made of peripheral noise under the assumption (usually made implicitly) that there is no central noise. For example, under what conditions (if any) can one ignore central noise and still obtain reliable estimates of the degree of correlation between peripheral channels? In the following paragraphs, we report the results of some preliminary illustrative computations relevant to such questions.

Assume, as illustrated schematically in Fig. 3, that the central noise can be described by adding an independent Gaussian noise of mean zero and variance σ_C^2 to the decision variable or, equivalently, to the decision criterion, where the decision variable is specified as the logarithm of the likelihood ratio. (Note that since the decision axis is dimensionless, the quantity σ_C is dimensionless.) In this case, Eq. (6) generalizes to[5]

$$d' = \frac{1}{\sigma}\,\frac{\left[\sum_{j=1}^{N}\Delta_j^2 - \frac{\sigma_R^2}{\sigma^2 + N\sigma_R^2}\left(\sum_{j=1}^{N}\Delta_j\right)^2\right]}{\left[\sum_{j=1}^{N}\Delta_j^2 - \frac{\sigma_R^2}{\sigma^2 + N\sigma_R^2}\left(\sum_{j=1}^{N}\Delta_j\right)^2 + \sigma_C^2\sigma^2\right]^{1/2}}. \qquad (16)$$

Again assuming for simplicity that $\Delta_j = \Delta$ in the level discrimination experiment and that $\Sigma\Delta_j = 0$ and $\Delta_j = \pm\Delta$ in the shape discrimination experiment, one obtains the following generalizations of Eqs. (12), (14), and (15):

$$d'_L = \frac{\sqrt{N}\,\Delta/\sqrt{\sigma^2 + N\sigma_R^2}}{\sqrt{1 + \sigma_C^2(\sigma^2 + N\sigma_R^2)/N\Delta^2}}, \qquad (17)$$

$$d'_S = \frac{\sqrt{N}\,\Delta/\sigma}{\sqrt{1 + \sigma_C^2\sigma^2/N\Delta^2}}, \qquad (18)$$

$$\left(\frac{d'_S}{d'_L}\right)^2 = \left(1 + N\frac{\sigma_R^2}{\sigma^2}\right)\left[1 + \left(\frac{\sigma^2}{N\sigma_R^2} + \frac{\Delta^2}{\sigma_C^2\sigma_R^2}\right)^{-1}\right]. \qquad (19)$$

These equations have a number of interesting implications. For example, they imply that d' varies as Δ when Δ is large, but as Δ^2 when Δ is small (assuming that σ_C is independent of Δ).[6] Note also that examination of the function $d'(\Delta)$ for both large and small Δ and for both level and shape discrimination permits determination of the variances σ_R^2 and σ_C^2 once σ^2 is known. Note, finally, that if one attempts to estimate the peripheral variance without taking account of the central variance (i.e., if it is assumed incorrectly that $\sigma_C = 0$ in making this estimate), then the peripheral variance will be overestimated and mistakenly appear to depend on the variables N and Δ, and the ratio σ_R/σ of the correlated to uncorrelated components will be overestimated.[7] A graphical display of Eq. (19) is shown in Fig. 4.

In the above formulation, the central processor takes no account of the central noise: the noise is added after the decision variable is formed. If the central processor were aware of the central noise, and σ_C were a constant, then the central processor could eliminate the effect of σ_C merely by per-

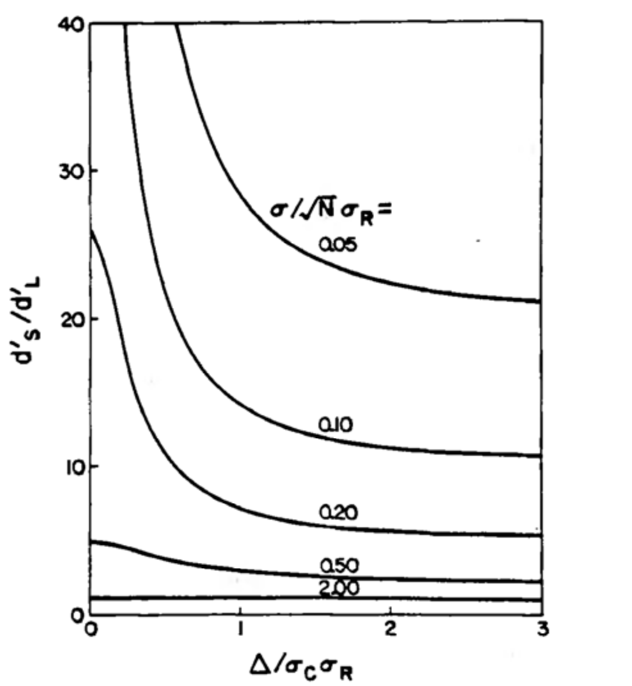

FIG. 4. The dependence of d'_S/d'_L on $\Delta/\sigma_C\sigma_R$ and $\sigma/\sqrt{N}\,\sigma_R$ as specified by Eq. (19).

forming a monotonic stretching transformation of the decision axis.[8] We now consider a different formulation in which the central noise is added before the decision variable is formed and the processing takes account of the central noise (like the peripheral noise) in an ideal manner.

Assume now, as pictured in Fig. 5, that the central noise can be described by adding independent Gaussian noises to the variables X_j to form the new variables $\widetilde{X}_j$. Assume, furthermore, that these noises have zero mean and a common variance σ_H^2 independent of j. The means and variances of the $\widetilde{X}_j$ are then given by the results shown in Eqs. (10) and (11) for the X_j, modified by the addition of σ_H^2 to the expressions for the variance. Making use of the statistical independence of $\widetilde{X}_1,...,\widetilde{X}_N$, we obtain[9] in place of Eq. (16)

$$d' = \left\{ \frac{(\Sigma_{j=1}^{N}\Delta_j)^2}{N(\sigma^2 + N\sigma_R^2) + \sigma_H^2} + \sum_{j=2}^{N}\left[\left(\frac{j-1}{j}\right) \times \frac{[\Delta_j - (j-1)^{-1}\Sigma_{k=1}^{j-1}\Delta_k]^2}{\sigma^2+\sigma_H^2}\right]\right\}^{1/2}. \quad (20)$$

When $\sigma_H = 0$, this formula for d' reduces to that given by Eq. (6) (a fact that necessarily follows from ideal-receiver theory, but is tedious to show directly).

Assuming once again that $\Delta_j = \Delta$ in the level-discrimination experiment and that $\Sigma\Delta_j = 0$ in the shape-discrimination experiment, one obtains in place of Eqs. (17) and (18)

$$d'_L = \frac{\sqrt{N}\Delta/\sqrt{\sigma^2 + N\sigma_R^2}}{\sqrt{1 + (\sigma_H^2/N)/(\sigma^2 + N\sigma_R^2)}}, \quad (21)$$

$$d'_S = \left\{\sum_{j=2}^{N}\left[\left(\frac{j-1}{j}\right) \times \frac{[\Delta_j - (j-1)^{-1}\Sigma_{k=1}^{j-1}\Delta_k]^2}{\sigma^2+\sigma_H^2}\right]\right\}^{1/2}. \quad (22)$$

Further insight can be obtained by now assuming in addition that $\Delta_j = \pm\Delta$ in the shape experiment [as we did for Eq. (18)] and that $N = 2$ for both experiments [since Eq. (22) remains complicated if N is not restricted]. With these additional assumptions, we obtain

$$d'_L = \frac{\sqrt{2}\Delta/\sqrt{\sigma^2 + 2\sigma_R^2}}{\sqrt{1 + (\sigma_H^2/2)/(\sigma^2 + 2\sigma_R^2)}}, \quad (23)$$

$$d'_S = \frac{\sqrt{2}\Delta/\sigma}{\sqrt{1 + \sigma_H^2/\sigma^2}}, \quad (24)$$

$$\left(\frac{d'_S}{d'_L}\right)^2 = 1 + \frac{2\sigma_R^2/\sigma_H^2 - \frac{1}{2}}{1 + \sigma^2/\sigma_H^2}. \quad (25)$$

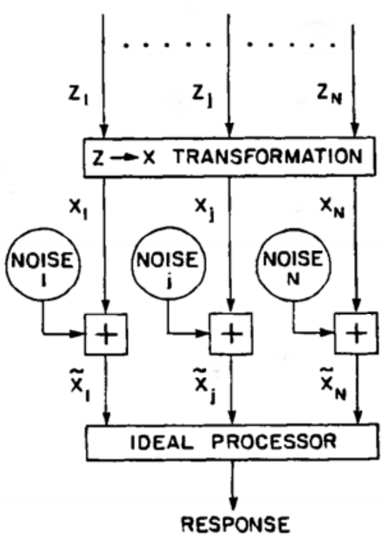

FIG. 5. Central noise case 2. In this model, the central processing is degraded by adding independent Gaussian noises to the transformed variables X_j.

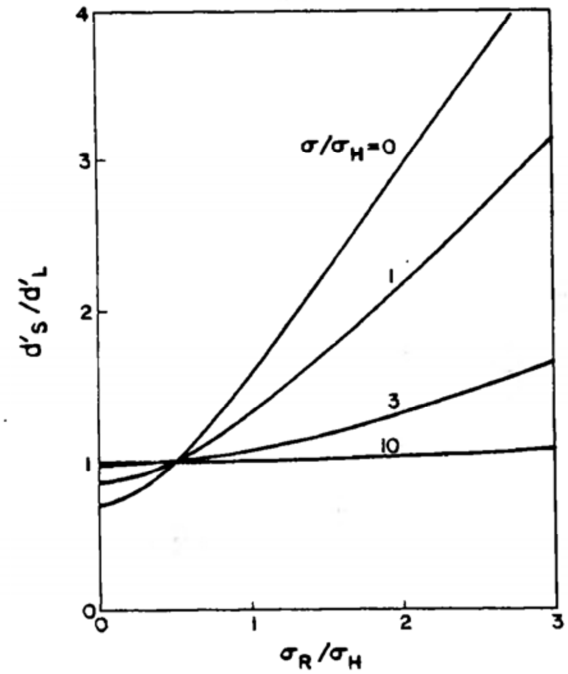

FIG. 6. The dependence of d'_S/d'_L on σ_R/σ_H and σ/σ_H as specified by Eq. (25).

These results differ in a variety of ways from those obtained with the criterion-noise assumption. For example, d' is now predicted to be linear in Δ for all ranges of Δ, and d'_S/d'_L is predicted to be independent of Δ. Also, the variance of the central noise now multiples the *reciprocal* of the variance of the peripheral noise rather than the variance of the peripheral noise itself [e.g., Eq. (21) has $\sigma_H^2/(\sigma^2 + N\sigma_R^2)$ whereas Eq. (17) has $\sigma_C^2(\sigma^2 + N\sigma_R^2)$]. In addition, ignoring the central noise now leads to an *underestimate* of the ratio σ_R/σ rather than an overestimate. Finally, one is no longer necessarily constrained to the condition $d'_S > d'_L$: one can now have $d'_S < d'_L$. Provided $\sigma_R^2 > \sigma_H^2/4$, the results on d' with this form of central noise will be the same as those obtained without any central noise but with the peripheral noise components altered as follows: $\sigma^2 \rightarrow \sigma^2 + \sigma_H^2$, $\sigma_R^2 \rightarrow \sigma_R^2 - \sigma_H^2/4$. Clearly, different types of central noise can lead to very different types of results. A graphical display of Eq. (25) is shown in Fig. 6.

V. CONCLUDING REMARKS

Clarification of the issues discussed in this paper requires both further analysis and further experimentation.

Additional theoretical efforts are needed to explore central noise structures and to determine how these structures interact with estimates of peripheral noise components. The cases considered in this paper are merely illustrative. In conjunction with these theoretical efforts, comparisons should be made between the central noise models and the data on response variability in situations where it can safely be assumed that the observed variability is due primarily to central factors (e.g., memory limitations or sequential effects).

Further attention should also be given to interchannel correlation and the way in which discrimination sensitivity changes with stimulus bandwidth W. On the whole, data in the literature (e.g., Moore and Raab, 1975; Buus and Florentine, 1981; Klumpp and Eady, 1956; Gabriel and Colburn, 1981) suggest that the increase in d' with an increase in W is significantly slower than what would be expected ($\sqrt{W}$) if the effective interchannel correlation were zero. Detailed analyses of these data, combined with computations for covariance structures in which the correlation between frequency channels decreases as the frequency separation between channels increases, could prove illuminating.

Experiments comparing discrimination of level and discrimination of spectral shape (using common subjects, procedures, stimuli and increments) can be used to determine variance ratios. If such experiments show that $d'_S < d'_L$, many models can be eliminated.

Empirical study of the Lim–Green discrepancy should be pursued by performing both types of tests with common subjects and procedures. If this study shows that frequency separation degrades resolution only when resolution is limited by memory factors, then further experiments should be performed in which the memory load is varied systematically.

Application of the roving-level discrimination paradigm to the determination of critical bandwidth W_C might also prove rewarding. In the conventional fixed-level, critical-band masking experiment, W_C is determined from the function $T(W)$ describing the dependence of the threshold T of a narrow-band signal on the bandwidth W of the masking noise (with spectral level of the masking noise held fixed). The threshold T tends to increase with W for $W < W_C$ and remain constant for $W \gg W_C$. In the roving-level version of this paradigm, where the useful cues concern spectral shape, T should *decrease* with W for $W < W_C$ and then remain flat for $W \gg W_C$. Comparison of the results of the roving-level masking experiment (which bears many similarities to the experiment by Hall *et al.*, 1984, on "comodulation masking release") to the results of the fixed-level masking experiment might provide a sharper definition of the critical band.

Finally, it should be noted that the idea of using roving levels to eliminate intensity cues and force the listener to discriminate on the basis of spectral *shape* can be naturally extended to a variety of other properties of the spectrum. For example, one can rove the mean slope of the spectrum as well as the level and thereby force the listener to discriminate solely on the basis of shape characteristics that are revealed through local variations in slope.

In conclusion, we again stress that all the remarks in this paper apply to interaural phase and interaural amplitude as well monaural amplitude. Thus all the suggested analyses and experiments apply to the interaural domain as well as the monaural domain.

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APPENDIX A

Sensitivity d' is given by the matrix equation

$$(d')^2 = [\Delta_1,...,\Delta_N]G[\Delta_1,...,\Delta_N]^T,$$

where G is the inverse of the covariance matrix K (e.g., van Trees, 1968, p. 98). In our application, the matrices $K = [k_{ij}]$ and $G = [g_{ij}]$ are given by

$$k_{ii} = \sigma_i^2 + \sigma_R^2, \quad k_{ij} = \sigma_R^2 \quad (i \neq j),$$

$$g_{ii} = \frac{1}{\sigma_i^2} - \frac{F}{\sigma_i^4}, \quad g_{ij} = -\frac{F}{\sigma_i^2\sigma_j^2} \quad (i \neq j),$$

where

$$F = \frac{\sigma_R^2}{1 + \sigma_R^2 \Sigma_{j=1}^N 1/\sigma_j^2}.$$

That G is, in fact, the inverse of K can be seen by showing that $P = GK$ is equal to the identity matrix. The diagonal terms of P are given by

$$p_{ii} = \sum_{j=1}^N k_{ij}g_{ji} = k_{ii}g_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^N k_{ij}g_{ji}.$$

However,

$$k_{ii}g_{ii} = (\sigma_i^2 + \sigma_R^2)\left(\frac{1}{\sigma_i^2} - \frac{F}{\sigma_i^4}\right)$$

and

$$\sum_{\substack{j=1 \\ j\neq i}}^N k_{ij}g_{ji} = -\sum_{\substack{j=1 \\ j\neq i}}^N \frac{F\sigma_R^2}{\sigma_i^2\sigma_j^2}.$$

Thus

$$p_{ii} = (\sigma_i^2 + \sigma_R^2)\left(\frac{1}{\sigma_i^2} - \frac{F}{\sigma_i^4}\right) - \sum_{\substack{j=1 \\ j\neq i}}^N \frac{F\sigma_R^2}{\sigma_i^2\sigma_j^2}$$

$$= 1 + \frac{\sigma_R^2}{\sigma_i^2} - \frac{F}{\sigma_i^2}\left(1 + \sigma_R^2 \sum_{j=1}^N \frac{1}{\sigma_j^2}\right) = 1.$$

The off-diagonal terms are given by

$$p_{ij} = \sum_{r=1}^N k_{ir}g_{rj} = k_{ii}g_{ij} + k_{ij}g_{jj} + \sum_{\substack{r=1 \\ r\neq i,j}}^N k_{ir}g_{rj}$$

$$= -\frac{F(\sigma_i^2+\sigma_R^2)}{\sigma_i^2\sigma_j^2} + \sigma_R^2\left(\frac{1}{\sigma_j^2} - \frac{F}{\sigma_j^4}\right) - \frac{F\sigma_R^2}{\sigma_j^2}\sum_{\substack{r=1\\ r\neq i,j}}^{N}\frac{1}{\sigma_r^2}$$

$$= \frac{1}{\sigma_j^2}\left[\sigma_R^2 - F\left(1+\sigma_R^2\sum_{r=1}^{N}\frac{1}{\sigma_r^2}\right)\right] = 0.$$

Hence P is the identity matrix.

Equation (3) can now be derived as follows. Let

$$[\Gamma_1,...,\Gamma_N] = [\Delta_1,...,\Delta_N]G.$$

Then

$$\Gamma_j = \sum_{i=1}^{N}\Delta_i g_{ij} = \Delta_j g_{jj} + \sum_{\substack{i=1\\ i\neq j}}^{N}\Delta_i g_{ij}$$

$$= \Delta_j\left(\frac{1}{\sigma_j^2} - \frac{F}{\sigma_j^4}\right) - \sum_{\substack{i=1\\ i\neq j}}^{N}\frac{F\Delta_i}{\sigma_i^2\sigma_j^2}.$$

Thus

$$(d')^2 = [\Gamma_1,...,\Gamma_N][\Delta_1,...,\Delta_N]^T = \sum_{j=1}^{N}\Gamma_j\Delta_j$$

$$= \sum_{j=1}^{N}\left[\Delta_j^2\left(\frac{1}{\sigma_j^2} - \frac{F}{\sigma_j^4}\right) - \sum_{\substack{i=1\\ i\neq j}}^{N}\frac{F\Delta_i\Delta_j}{\sigma_i^2\sigma_j^2}\right]$$

$$= \sum_{j=1}^{N}\left(\frac{\Delta_j^2}{\sigma_j^2} - F\sum_{i=1}^{N}\frac{\Delta_i\Delta_j}{\sigma_i^2\sigma_j^2}\right)$$

$$= \sum_{j=1}^{N}\left(\frac{\Delta_j}{\sigma_j}\right)^2 - F\left(\sum_{j=1}^{N}\frac{\Delta_j}{\sigma_j^2}\right)^2.$$

APPENDIX B

Since the X_j are linear combinations of the Z_j, the X_j are Gaussian. Thus to demonstrate that the X_j are statistically independent, it is sufficient to show they are uncorrelated.

Consider first the case of the variables X_a and X_b with $1<a<b$. Denoting the mean (conditioned on either S_1 or S_2) by a bar, one has

$$\overline{(X_a - \overline{X_a})(X_b - \overline{X_b})}$$

$$= \sqrt{\frac{a-1}{a}}\sqrt{\frac{b-1}{b}}\ \overline{\left((Z_a - \overline{Z_a}) - \frac{1}{a-1}\sum_{k=1}^{a-1}(Z_k - \overline{Z_k})\right)\left((Z_b - \overline{Z_b}) - \frac{1}{b-1}\sum_{q=1}^{b-1}(Z_q - \overline{Z_q})\right)}$$

$$= \sqrt{\frac{a-1}{a}}\sqrt{\frac{b-1}{b}}\left(\overline{(Z_a - \overline{Z_a})(Z_b - \overline{Z_b})} + \sum_{k=1}^{a-1}\sum_{q=1}^{b-1}\frac{\overline{(Z_k - \overline{Z_k})(Z_q - \overline{Z_q})}}{(a-1)(b-1)} - \sum_{k=1}^{a-1}\frac{\overline{(Z_b - \overline{Z_b})(Z_k - \overline{Z_k})}}{a-1} - \sum_{q=1}^{b-1}\frac{\overline{(Z_a - \overline{Z_a})(Z_q - \overline{Z_q})}}{b-1}\right)$$

$$= \sqrt{\frac{a-1}{a}}\sqrt{\frac{b-1}{b}}\left(\sigma_R^2 + \frac{(a-1)\sigma^2 + (a-1)(b-1)\sigma_R^2}{(a-1)(b-1)} - \frac{(a-1)\sigma_R^2}{(a-1)} - \frac{(b-1)\sigma_R^2 + \sigma^2}{(b-1)}\right) = 0.$$

Thus, X_a and X_b are uncorrelated for $1<a<b$.

Consider next the case of X_a and X_1

$$\overline{(X_a - \overline{X_a})(X_1 - \overline{X_1})} = \overline{\left(\sum_{q=1}^{N}(Z_q - \overline{Z_q})\right)\sqrt{\frac{a-1}{a}}\left((Z_a - \overline{Z_a}) - \frac{1}{a-1}\sum_{k=1}^{a-1}(Z_k - \overline{Z_k})\right)}$$

$$= \sqrt{\frac{a-1}{a}}\left(\sum_{q=1}^{N}\overline{(Z_a - \overline{Z_a})(Z_q - \overline{Z_q})} - \frac{1}{a-1}\sum_{q=1}^{N}\sum_{k=1}^{a-1}\overline{(Z_q - \overline{Z_q})(Z_k - \overline{Z_k})}\right)$$

$$= \sqrt{\frac{a-1}{a}}\left(N\sigma_R^2 + \sigma^2 - \frac{1}{a-1}[N(a-1)\sigma_R^2 + (a-1)\sigma^2]\right) = 0.$$

Thus X_a and X_1 are uncorrelated.

APPENDIX C

The equations for the means are obvious. The equations for the variances can be derived as follows:

$$V(X_1) = \overline{(X_1 - \overline{X_1})^2} = \overline{\left(\sum_{j=1}^{N}Z_j - \sum_{j=1}^{N}M_{ij}\right)^2} = \overline{\left(\sum_{j=1}^{N}(Z_j - M_{ij})\right)^2}$$

$$= \sum_{j=1}^{N}\overline{(Z_j - M_{ij})^2} + \sum_{j=1}^{N}\sum_{\substack{k=1\\ k\neq j}}^{N}\overline{(Z_j - M_{ij})(Z_k - M_{ik})}$$

$$= N(\sigma^2 + \sigma_R^2) + N(N-1)\sigma_R^2 = N(\sigma^2 + N\sigma_R^2).$$

For $j \geqslant 2$,

$$V(X_j) = \overline{(X_j - \overline{X_j})^2} = \overline{\left[\sqrt{\frac{j-1}{j}}\left(Z_j - \frac{1}{j-1}\sum_{k=1}^{j-1} Z_k\right) - \sqrt{\frac{j-1}{j}}\left(M_{ij} - \frac{1}{j-1}\sum_{k=1}^{j-1} M_{ik}\right)\right]^2}$$

$$= \frac{j-1}{j}\overline{\left((Z_j - M_{ij}) - \frac{1}{j-1}\sum_{k=1}^{j-1}(Z_k - M_{ik})\right)^2}$$

$$= \frac{j-1}{j}\left(\overline{(Z_j - M_{ij})^2} - \frac{2}{j-1}\sum_{k=1}^{j-1}\overline{(Z_k - M_{ik})(Z_j - M_{ij})}\right.$$

$$\left.\times + \frac{1}{(j-1)^2}\sum_{q=1}^{j-1}\sum_{k=1}^{j-1}\overline{(Z_k - M_{ik})(Z_q - M_{iq})}\right)$$

$$= \left(\frac{j-1}{j}\right)\left((\sigma^2 + \sigma_R^2) - \frac{2(j-1)\sigma_R^2}{j-1} + \frac{(j-1)(\sigma^2 + \sigma_R^2) + (j-1)(j-2)\sigma_R^2}{(j-1)^2}\right) = \sigma^2.$$

[1]Data on the effect of duration on intensity jnds are available in Henning and Psotka, 1969; Henning, 1970; and Florentine, 1986. If the data are processed to factor out the effects of changes in energy level, the dependence on duration, like the dependence on level, is seen to be relatively weak.

[2]Detailed quantitative application of our model to these experiments would have to take account of the fact that the probability density of R is assumed to be Gaussian in the model but is not Gaussian in the experiments (at the input to the system it is rectangular).

[3]The existence of many distinct linear transformations of the Z_j which produce statistically independent variables X_j is a consequence of the special structure of the covariance matrix for the Z_j, i.e., the equality of the off-diagonal terms. However, all such transformations will contain a mean-level variable like X_1.

[4]In actually performing such comparative experiments, one would have to take account of the finding (Green, personal communication) that substantial retraining is required to achieve optimal performance when the task switches back and forth between level discrimination and spectral-shape discrimination. Apparently, switching one's processing back and forth between the two tasks is not as simple as our model (using the X_j variables as a display) implies.

[5]Equation (16) can be derived by noting that (a) the addition of the criterion noise has no effect on the mean of the decision variable, but merely increases the variance σ_D^2 of the decision variable to $\sigma_D^2 + \sigma_C^2$ and (b) for the case in which there is no criterion noise, $d' = \sigma_D$ (e.g., van Trees, 1968, p. 99). Letting Δm denote the difference between the means of the decision variables for the hypotheses S_1 and S_2 and an asterisk denote the inclusion of criterion noise, one has

$$(d'_*)^2 = \frac{(\Delta m_*)^2}{\sigma_C^2 + \sigma_D^2} = \frac{(\Delta m)^2}{\sigma_C^2 + \sigma_D^2} = \frac{(d')^2\sigma_D^2}{\sigma_C^2 + \sigma_D^2} = \frac{(d')^4}{\sigma_C^2 + (d')^2}.$$

Thus $d'_* = (d')^2/\sqrt{\sigma_C^2 + (d')^2}$. Equation (16) is obtained by substituting for d' the expression given by Eq. (6).

[6]This result for $d'(\Delta)$ is valid provided only that the Δ_j in Eq. (16) satisfy the relations $\Delta_j = A_j\Delta$ for some set of constants $A_1,\ldots,A_N$.

[7]Let $\overline{\sigma_R^2/\sigma^2}$ denote the estimate of σ_R^2/σ^2 obtained from Eq. (19) under the assumption that $\sigma_C^2 = 0$. Then

$$\overline{\sigma_R^2/\sigma^2} = \frac{1}{N}\left[\left(\frac{d'_S}{d'_L}\right)^2 - 1\right] = \frac{\sigma_R^2}{\sigma^2} + \frac{\sigma_C^2\sigma_R^2}{N\Delta^2 + \sigma_C^2\sigma^2} + \frac{\sigma_R^2}{\sigma^2}\left(\frac{N\sigma_C^2\sigma_R^2}{N\Delta^2 + \sigma_C^2\sigma^2}\right) > \frac{\sigma_R^2}{\sigma^2}.$$

[8]Since it seems obvious, on intuitive grounds, that noise added after formation of the decision variable, such as criterion noise, actually does have a degrading effect on performance, perhaps one should conclude here that σ_C is not a constant but grows linearly with stretching of the decision axis. Certainly, the idea that the amount of criterion noise grows with the range of experimental stimuli presented is not a new one (e.g., see Lockhead, 1973).

[9]This result follows directly from Eq. (2) with $\widetilde{\Delta}_j$ substituted for Δ_j and $\tilde{\sigma}_j$ substituted for σ_j, where $\widetilde{\Delta}_j$ is the difference between the means and $\tilde{\sigma}_j^2$ is the variance of the variable $\widetilde{X}_j$. From Eqs. (10) and (11), modified by the addition of the variance term σ_H^2, one has $\tilde{\sigma}_1^2 = N(\sigma^2 + N\sigma_R^2) + \sigma_H^2$ and $\tilde{\sigma}_j^2 = \sigma^2 + \sigma_H^2$ for $2 \leqslant j \leqslant N$. Similarly, $\widetilde{\Delta}_1^2 = (\Sigma_{k=1}^N \Delta_k)^2$ and $\widetilde{\Delta}_j^2 = \{\sqrt{1 - 1/j}[\Delta_j - (j-1)^{-1}\Sigma_{k=1}^{j-1}\Delta_k]\}^2$ for $2 \leqslant j \leqslant N$.

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